

Modality A: Midterm exam (sample 2, key)

1. A set $\{\phi_i(t)\}$ of waveforms is orthonormal if $\langle \phi_i, \phi_k \rangle = \delta_{ik}$: every waveform has unit energy and any two distinct waveforms are orthogonal.

(a) True ★
(b) False

Solution: Statement 1, definition of an orthonormal set.

2. By Parseval's identity, an inner product can be computed in the time or in the frequency domain,

$$\langle x, y \rangle = \int_{-\infty}^{\infty} \hat{x}(f) \hat{y}^*(f) df,$$

so a signal has the same energy in both domains.

(a) True ★
(b) False

Solution: Statement 4, Parseval's identity.

3. A rectangle of amplitude A and width T centered at the origin has Fourier transform $A \operatorname{sinc}(Tf)$.

(a) True
(b) False ★

Solution: Statement 4. The transform of the rectangle is $AT \operatorname{sinc}(Tf)$. Its value at $f = 0$ is the area AT of the rectangle.

4. A baseband waveform of bandwidth 50 MHz allows at most 100 Msymbol/s and 800 Mbit/s with 256-QAM.

(a) True ★
(b) False

Solution: Statement 7: $R_s = 2W$ and $R_b = R_s \log_2 M$ with $\log_2 256 = 8$.

5. For a complex signal, $|x(t)|^2 = x(t)x^*(t)$ is the squared modulus, not the square, and the energy is $\int_{-\infty}^{\infty} |x(t)|^2 dt$.

(a) True ★
(b) False

Solution: Statement 8, energy of a complex signal.

6. The average energy per bit is $E_b = E_s \log_2 M$ for a one-to-one mapping of $\log_2 M$ bits onto M symbols.

(a) True
(b) False ★

Solution: Statement 10. The energy per bit is the symbol energy divided by the number of bits, $E_b = E_s / \log_2 M$.

7. In a Monte Carlo simulation, the relative frequency of an event approaches its probability, because the indicator of the event, equal to 1 when it occurs and 0 otherwise, has expectation equal to the probability.

(a) True ★
(b) False

Solution: Statement 11, law of large numbers applied to the indicator of an event.

8. The entropy of a discrete random variable is nonnegative, and it is additive for independent variables.

(a) True ★
(b) False

Solution: Statement 13, properties of the entropy.

9. A real Gaussian random variable with variance σ^2 has differential entropy $\frac{1}{2} \log_2(2\pi e \sigma^2)$, which depends on its mean.

(a) True
(b) False ★

Solution: Statement 15. The differential entropy of a Gaussian variable is $\frac{1}{2} \log_2(2\pi e \sigma^2)$ whatever its mean, since a shift does not change it (Statement 14).

10. The mutual information $I(X; Y) = H(Y) - H(Y|X)$ is nonnegative but not symmetric: in general $I(X; Y)$ differs from $I(Y; X)$.

(a) True
(b) False ★

Solution: Statement 16. The mutual information is nonnegative and symmetric.

11. As the bandwidth grows without bound with P and N_0 fixed, the capacity of the AWGN channel grows without bound as well.

(a) True
(b) False ★

Solution: Statement 20. The capacity approaches the finite limit $\frac{P}{N_0} \log_2 e$ bits per second.

12. On the Gaussian channel the ML detector chooses the constellation point nearest to the observation, and this nearest-point rule also holds with uniform, exponential or multiplicative noise.

(a) True
(b) False ★

Solution: Statement 22. The nearest-point rule is a property of Gaussian noise. With other noises the detector must compare the channel laws directly.