

Modality A: Midterm exam (sample 1, key)

1. The energy of a waveform is its inner product with itself, $E = \langle \phi, \phi \rangle$, a nonnegative real number.

(a) True ★
(b) False

Solution: Statement 1, energy as the inner product of a waveform with itself.

2. Two waveforms with disjoint supports are orthogonal.

(a) True ★
(b) False

Solution: Statement 2. Disjoint supports give a zero inner product.

3. A signal band-limited to $|f| \leq W$ belongs to the space spanned by the orthonormal sampling waveforms $\phi_i(t) = \sqrt{2W} \operatorname{sinc}(2W(t - \frac{i}{2W}))$, with coefficients $x_i = \sqrt{2W} x(\frac{i}{2W})$.

(a) True
(b) False ★

Solution: Statement 3. The coefficients are the samples scaled by $\frac{1}{\sqrt{2W}}$, not by $\sqrt{2W}$.

4. A passband transmission of bandwidth W and duration T provides WT orthonormal complex waveforms, that is, WT channel uses.

(a) True ★
(b) False

Solution: Statement 5, channel uses of a passband transmission.

5. Symbols sent as $\sum_{i=-\infty}^{\infty} X_i \phi(t - iT)$ are free of inter-symbol interference when the shifted waveforms $\phi(t - iT)$ are orthogonal, which holds if and only if the similarity function $R_{\phi\phi}(\tau)$ vanishes at every nonzero multiple of T .

(a) True ★
(b) False

Solution: Statement 6, orthogonality of the shifted waveforms.

6. For equiprobable symbols, the average symbol energy is the arithmetic mean of the squared moduli of the points, excluding any point at the origin, which costs no energy.

(a) True
(b) False ★

Solution: Statement 9. A point at the origin costs no energy but counts in the average.

7. In MATLAB, $(\text{rand} < p)$ is a bit equal to 1 with probability p , for $0 \leq p \leq 1$.

(a) True ★
(b) False

Solution: Statement 12: $\Pr(\text{rand} < p) = p$.

8. The differential entropy of a continuous random variable is nonnegative, like the entropy of a discrete one.

(a) True
(b) False ★

Solution: Statement 14. The differential entropy can be negative, for instance $\log_2 a < 0$ for a uniform variable on an interval of length $a < 1$.

9. For the binary erasure channel with erasure probability ϵ , the mutual information with equiprobable inputs is $1 - \epsilon$ bits per channel use.

(a) True ★
(b) False

Solution: Statement 17, mutual information of the BEC.

10. The capacity of the real AWGN channel with average symbol energy E_s and noise variance σ^2 is $\log_2(1 + \frac{E_s}{\sigma^2})$ bits per channel use.

(a) True
(b) False ★

Solution: Statement 19. For the real channel the capacity is $\frac{1}{2} \log_2(1 + \frac{E_s}{\sigma^2})$; the expression without the $\frac{1}{2}$ is the complex one.

11. Where two likelihoods tie, the maximum likelihood detector decides by the prior, whereas the maximum a posteriori detector may decide arbitrarily.

(a) True
(b) False ★

Solution: Statement 21. It is the other way round: the MAP detector decides by the prior and the ML detector may decide arbitrarily.

12. The shortest reversible mapping of groups of k bits onto sequences of symbols of an M -point constellation carries exactly $\log_2 M$ bits per symbol.

(a) True
(b) False ★

Solution: Statement 24. It carries $\frac{k}{n} \leq \log_2 M$ bits per symbol, with equality only when 2^k is a power of M . For instance, 10 bits need 7 symbols of 3-PSK, $\frac{10}{7} < \log_2 3$.